The experiments yielded the following interesting result: If the flow of liquid is additionally made turbulent in the forechamber of the nozzle (by inserting a 1 mm in diameter wire into it perpendicular to the axis), then a dip appears in the measured distributions at the center of the spray cone. This probably caused by the fact that turbulence, which results in breakup of primarily the most turbulent volumes of the liquid, plays a significant role in the breakup of the jet.

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FORMULATION OF PROBLEMS WITH MOVING PHASE-TRANSITION BOUNDARIES IN HYDROTHERMAL STRATA

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The study of heat and mass transfer processes in hydrothermal strata is of unquestionable interest for the design of systems for extracting geothermal energy, an alternative energy resource, as well as for a better understanding of the dynamics of thermal processes in the lithosphere. Hydrothermal systems are natural strata saturated with hot water, steam, or a water—steam mixture. A change in the external conditions (e.g., a change in the heat flux of the Earth or drilling of a well into the formation) results in filtration flow of water (steam), accompanied by water—steam phase transitions and a change in the temperature, pressure, and phase composition of the formation. The mathematical description is constructed on the basis of the laws of conservation of mass and energy and Darcy's law for two-phase nonisothermal filtration. The model is closed by using the phase diagram for water. The known models of heat and mass transfer in hydrothermal strata [1-4] have been formulated in terms of pressure and enthalpy. The water saturation in the case of a steam—water mixture is calculated from the water and steam densities and the enthalpies of the water, steam, and steam—water mixture. The use of enthalpy as the sought function presumes that the system of conservation equations is closed with the p—h diagram of water. In fact, this model, a realization of the well-known enthalpy approach to the description of phase transitions [5], does not explicitly contain a moving phase boundary, i.e., a surface of a strong discontinuity of the water saturation function. The approach described above, however, has some shortcomings. The boundary conditions must be given in terms of the enthalpy, which is difficult to do in the case of boundary conditions of the first and third kinds. Moreover, the temperature field, which must be known for an understanding of the processes in the formation, remains undetermined.

In this paper a mathematical model of heat and mass transfer processes with phase transitions in hydrothermal strata, is formulated in terms of the temperature, pressure, and saturation and admits the existence of moving phase-transition surfaces, which are strong discontinuities of saturation function. The solution is constructed within the framework of the generalized solution of the Stefan problem [6], in many ways resembling analogous solutions of problems of the freezing and thawing of soils [7, 8] and the decomposition of gaseous hydrates in natural strata [9].

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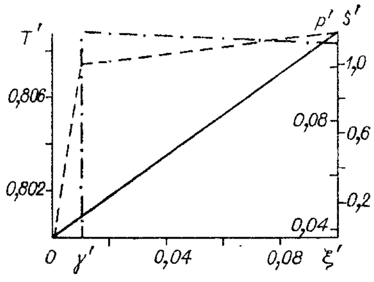

Fig. 1

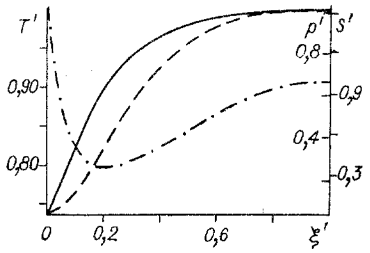

Fig. 2

Mathematical Formulation of the Problem. A hydrothermal system will be taken to mean a porous medium, saturated with water, steam, or a mixture of the two. The heat and mass transfer processes and phase transitions will be considered within the following approximation: the skeleton of the porous medium is incompressible and stationary, the steam is an ideal gas, the capillary effects are negligible, the local temperatures and pressures of the components of the medium coincide, and the viscosities are assumed to be functions of the temperature.

The principal equations include the laws of conservation of mass for water and steam

$$m\frac{\partial}{\partial t}S\rho_{w}+\operatorname{div}\rho_{w}\mathbf{v}_{w}=M,\quad m\frac{\partial}{\partial t}(1-S)\rho_{st}+\operatorname{div}\rho_{st}\mathbf{v}_{st}=-M;$$

the equations of Darcy's law

$$\mathbf{v}_{w}=-\frac{k}{\mu_{w}(T)}f_{w}(S)\operatorname{grad}p,\quad \mathbf{v}_{st}=-\frac{k}{\mu_{st}(T)}f_{st}(S)\operatorname{grad}p;$$

the states of the water and steam

$$\rho_{w}=\rho_{w0}[1+\alpha(p-p_{0})-\beta(T-T_{0})],\quad p=\rho_{st}RT;$$

the curve of the interface on the p–T diagram [10]

$$\lg p=A+B/T; \tag{1}$$

and the energy

$$\frac{\partial}{\partial t}(\rho e)_{eff}+\operatorname{div}(\rho_{w}h_{w}\mathbf{v}_{w}+\rho_{st}h_{st}\mathbf{v}_{st})=\operatorname{div}(\lambda_{eff}\operatorname{grad}T)$$
$$(\lambda_{eff}=mS\lambda_{w}+m(1-S)\lambda_{st}+(1-m)\lambda_{s},$$
$$(\rho e)_{eff}=mS\rho_{w}e_{w}+m(1-S)\rho_{st}e_{st}+(1-m)\rho_{s}e_{s}).$$

Here T is the temperature, p is the pressure, S is the water saturation, h is the enthalpy, ρ is the density, V is the filtration rate, M is the intensity of the phase transition, k is the permeability, f is the phase permeability, μ is the viscosity, α is the compressibility factor, β is the thermal expansion coefficient, λ is the thermal conductivity factor, m is the porosity; subscripts: w – water, st – steam, s – skeleton of the porous medium, and eff – effective value.

After identity transformations, with allowance for the known relation

$$dh=CdT+\frac{dp}{\rho}(1-\beta T)\quad\left(\beta=-\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right)$$

the initial system of equations reduces to a system of three equations in T, p, and S.

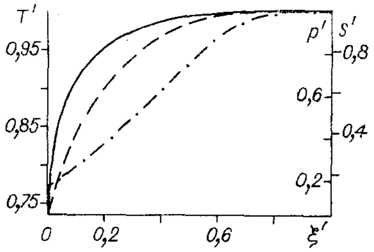

Fig. 3

Conditions at Moving Phase-Transition Boundaries. The initial and boundary conditions are given such that they are on either side of the equilibrium curve, in accordance with the phase diagram. In this case regions with different phase compositions should be separated by phase-transition surfaces, on which a strong discontinuity of the water saturation function is admitted. The conditions on the phase-transition surfaces follow from the laws of conservation of mass and energy at discontinuities [11].

$$[\rho(V_n-u_n)]_-^+=0,\quad [\rho h(V_n-u_n)-Q_n]_-^+=0,$$

where V is the velocity of the discontinuity, u is the velocity of the medium, and Q is the conductive heat flux; the subscript n denotes projection onto the normal.

Let us consider the following types of discontinuity:

1) the interface between the regions of steam (−) and steam-water mixture (+)

$$\rho_{st}\frac{k}{\mu_{st}}\,\mathrm{grad}\,p_- - k\left(\rho_{st}\frac{f_{st}}{\mu_{st}}+\rho_w\frac{f_w}{\mu_w}\right)\mathrm{grad}\,p_+ = mS_+(\rho_w-\rho_{st})V_n; \tag{2}$$

$$\lambda_+\,\mathrm{grad}\,T_+ - \lambda_-\,\mathrm{grad}\,T_- - kq\frac{\rho_w f_w}{\mu_w}\,\mathrm{grad}\,p_+ = mS_+q\rho_w V_n \tag{3}$$

($q = h_{st} - h_w$ is the heat of the phase transition);

2) the interface between the regions of the steam-water mixture (−) and water (+)

$$-\rho_w\frac{k}{\mu_w}\,\mathrm{grad}\,p_+ + k\left(\rho_{st}\frac{f_{st}}{\mu_{st}}+\rho_w\frac{f_w}{\mu_w}\right)\mathrm{grad}\,p_- = m(1-S_-)(\rho_w-\rho_{st})V_n,$$

$$\lambda_+\,\mathrm{grad}\,T_+ - \lambda_-\,\mathrm{grad}\,T_- - kq\frac{\rho_{st}f_{st}}{\mu_{st}}\,\mathrm{grad}\,p_- = m(1-S_-)q\rho_{st}V_n;$$

3) the interface between the regions of steam (−) and water (+)

$$\left(\rho_{st}\frac{k}{\mu_{st}}\,\mathrm{grad}\,p_- - \rho_w\frac{k}{\mu_w}\right)\mathrm{grad}\,p_+ = m(\rho_w-\rho_{st})V_n,$$

$$\lambda_+\,\mathrm{grad}\,T_+ - \lambda_-\,\mathrm{grad}\,T_- - kq\frac{\rho_{st}}{\mu_{st}}\,\mathrm{grad}\,p_- = mq\rho_w V_n.$$

Moreover, the continuity conditions for the temperature and pressure and the phase transition (1) are satisfied for all types of discontinuities.

We note that a functional relation between the pressure and temperature [phase equilibrium equation (2)] exists in the region of the steam-water mixture. This makes it possible to analyze the boundary conditions for the first and second types of discontinuities. The heat balance condition for a discontinuity of the first kind can be written as

$$\left(\lambda_+ - kqf_w\frac{\rho_w pB_*}{\mu_w T^2}\right)\mathrm{grad}\,T_+ - \lambda_-\,\mathrm{grad}\,T_- = mS_+q\rho_w V_n\quad (B_* = -2{,}3\,B). \tag{4}$$

On substituting the characteristic values of the parameters, we find that the term in parentheses in (4) vanishes at permeabilities $k \approx 10^{-18}$ m^2 and is negative at higher permeabilities. Since we are considering the extraction of heat carrier from a stratum, grad $T_+ > 0$ and grad $T_- > 0$. But then the right side should be negative at $k > 10^{-18}$ m^2, which means that the boundary conditions at the phase-transition surface cannot be satisfied and, therefore, that such a surface does not exist. Only at very low permeabilities, therefore, are solutions possible with moving saturation discontinuities and they do not exist over a wide range of parameters. Such discussions also hold for the other types of discontinuities.

Results of Numerical Calculations. The system of equations and the conditions at discontinuities, as formulated above, in the one-dimensional case admit self-similar solutions, which correspond to constant values of the initial and boundary conditions on a stationary surface (borehole wall) for T, p, and S. The original initial-value and boundary-value problems for the partial differential equations reduce to the boundary-value problems for systems of ordinary differential equations. The solution of the latter is determined numerically by the method of parallel ranging [12].

In the one-dimensional formulation we consider the development of hydrothermal reservoir, in which initially steam coexists with water in a state of thermodynamic equilibrium. The extraction of heat carrier from the stratum presupposes a pressure drop in the borehole. It is natural to expect that, depending on the extraction conditions and the stratum parameters, the extracted heat carrier can be either steam or steam–water mixture. Suppose that steam is extracted. Then an interface X(t) exists between the regions of steam and steam–water mixture; conditions (2) and (3) are satisfied at the interface.

$$\rho_{\text{st}}\frac{k}{\mu_{\text{st}}}\frac{\partial p}{\partial x_-} - k\left(\rho_{\text{st}}\frac{f_{\text{st}}}{\mu_{\text{st}}} + \rho_{\text{w}}\frac{f_{\text{w}}}{\mu_{\text{w}}}\right)\frac{\partial p}{\partial x_+} = mS_+(\rho_{\text{w}} - \rho_{\text{st}})\frac{dX}{dt},$$

$$\lambda_+\frac{\partial T}{\partial x_+} - \lambda_-\frac{\partial T}{\partial x_-} - kq\frac{\rho_{\text{w}}f_{\text{w}}}{\mu_{\text{w}}}\frac{\partial p}{\partial x_+} = mS_+q\rho_{\text{w}}\frac{dX}{dt}$$

$$(\lambda_+ = mS_+\lambda_{\text{w}} + m(1 - S_+)\lambda_{\text{st}} + (1 - m)\lambda_{\text{s}}, \quad \lambda_- = m\lambda_{\text{st}} + (1 - m)\lambda_{\text{s}}),$$

and so are the continuity conditions for the temperature and pressure and the phase transition

$$T_- = T_+ = T_*, \quad p_- = p_+ = p_*, \quad \lg p_* = A + B/T_*.$$

The water saturation function S here has a discontinuity from the sought value of S_+ to $S_- = 0$. The temperature $T = T^0$ and pressure $p = p^0$ are given at the stationary wall $x = 0$. The initial conditions have the form $T = T_0$, $p = p_0$, $S = S_0$, and $\log p_0 = A + B/T_0$ at $t = 0$.

Since the initial state is a state of thermodynamic equilibrium, the equations in the zone of the steam–water mixture should admit a change in the phase composition as a result of water–steam phase transitions. Analysis of the phase transition processes in such systems [7-9] shows that front model, which do not take volume phase transitions into account, are thermodynamically contradictory.

The equations in the zone of the steam–water mixture are written as

$$\begin{aligned}
&\left(1 - \frac{p}{\rho_{\text{w}0}RT}\right)\frac{\partial S}{\partial t} + \left(\alpha S + \frac{1 - S}{\rho_{\text{w}0}RT}\right)\frac{\partial p}{\partial t} - \left(\beta S + \frac{(1 - S)p}{\rho_{\text{w}0}RT^2}\right)\frac{\partial T}{\partial t} - \\
&- \frac{k}{m}\left(\frac{\alpha f_{\text{w}}}{\mu_{\text{w}}} + \frac{f_{\text{st}}}{\mu_{\text{st}}\rho_{\text{w}0}RT}\right)\left(\frac{\partial p}{\partial x}\right)^2 + \frac{k}{m}\Bigg(\frac{\beta f_{\text{w}}}{\mu_{\text{w}}} + \frac{f_{\text{st}}p}{\mu_{\text{st}}\rho_{\text{w}0}RT^2} + \\
&+ \frac{f_{\text{st}}p}{\mu_{\text{st}}^2\rho_{\text{w}0}RT}\frac{d\mu_{\text{st}}}{dT} + \frac{f_{\text{w}}}{\mu_{\text{st}}^2}\frac{d\mu_{\text{w}}}{dT}\Bigg)\frac{\partial p}{\partial x}\frac{\partial T}{\partial x} - \frac{k}{m}\left(\frac{1}{\mu_{\text{w}}}\frac{df_{\text{w}}}{dS} + \frac{p}{\mu_{\text{st}}\rho_{\text{w}0}RT}\frac{df_{\text{st}}}{dS}\right)\frac{\partial S}{\partial x}\frac{\partial T}{\partial x} = \\
&= \frac{k}{m}\left(\frac{f_{\text{w}}}{\mu_{\text{w}}} + \frac{f_{\text{st}}p}{\mu_{\text{st}}\rho_{\text{w}0}RT}\right)\frac{\partial^2 p}{\partial x^2},
\end{aligned} \qquad (5)$$

$$\left(\frac{(\rho C)_{\mathrm{eff}}}{m} - (1-S)\frac{pq}{RT^2}\right)\frac{\partial T}{\partial t} + \left(S(1-\beta T) + \frac{q(1-S)}{RT} - 1\right)\frac{\partial p}{\partial t} -$$

$$- \frac{pq}{RT}\frac{\partial S}{\partial t} - \frac{k}{m}\left(\frac{f_{\mathrm{w}}}{\mu_{\mathrm{w}}}(1-\beta T) + \frac{f_{\mathrm{st}} q}{\mu_{\mathrm{st}} RT}\right)\left(\frac{\partial p}{\partial x}\right)^2 +$$

$$+ \frac{k}{m}\left(\frac{f_{\mathrm{st}} p}{\mu_{\mathrm{st}} RT}(q/T - C_p) - \rho_{\mathrm{w}0} C_{\mathrm{w}} f_{\mathrm{w}}/\mu_{\mathrm{w}}\right)\frac{\partial p}{\partial x}\frac{\partial T}{\partial x} - \lambda_{\mathrm{w}}\frac{\partial S}{\partial x}\frac{\partial T}{\partial x} - \tag{6}$$

$$- \frac{qpk}{mRT\mu_{\mathrm{st}}}\frac{df_{\mathrm{st}}}{dS}\frac{\partial S}{\partial x}\frac{\partial p}{\partial x} = \frac{kqf_{\mathrm{st}} p}{mR\mu_{\mathrm{st}} T}\frac{\partial^2 p}{\partial x^2} + \frac{\lambda_{\mathrm{eff}}}{m}\frac{{}^2 T}{\partial x^2};$$

$$\lg p = A + B/T. \tag{7}$$

The effective heat capacity here is $(\rho C)_{\mathrm{eff}} = mS\rho_{\mathrm{w}}C_{\mathrm{w}} + m(1-S)\rho_{\mathrm{st}}C_{\mathrm{p}} + (1-m)\rho_{\mathrm{s}}C_{\mathrm{s}}$. The equations in the zone of the steam flow from (5) and (6), if we set $S = 0$.

The problem formulated at constant T_0, p_0, S_0, T^0, and p^0 has a self-similar solution of the form $T = T(\xi)$, $p = p(\xi)$, $S = S(\xi)$, $X = \gamma t^{1/2}$ and $\xi = xt^{-1/2}$, whose determination reduces to numerical solution of the boundary-value problem for a system of boundary differential equations. In the calculations we used the approximation of the empirical data for the functions $f_{\mathrm{w}}(S)$, $f_{\mathrm{st}}(S)$, $\mu_{\mathrm{w}}(T)$, $\mu_{\mathrm{st}}(T)$, [10, 13] and the following values of the parameters: $\rho_{\mathrm{w}0} = 1000$ kg/m³, $\rho_{\mathrm{s}} = 2000$ kg/m³, $\lambda_{\mathrm{w}} = 0.58$ W/m·K, $\lambda_{\mathrm{s}} = 2.09$ W/m·K, $\lambda_{\mathrm{st}} = 0.02$ W/m·K, $C_{\mathrm{w}} = 4.19$ kJ/kg·K, $C_{\mathrm{s}} = 0.9$ kJ/kg·K, $Cp = 1.8$ kJ/kg·K, $\alpha = 0.49 \cdot 10^{-9}$ Pa^{-1}, $\beta = 0.001$ K^{-1}, $m = 0.25$, $R = 461$ J/kg·K, $q = 1900$ kJ/kg, $\mu_{\mathrm{st}0} = 1.2 \cdot 10^{-5}$ Pa·sec, $A = 5.44$, and $B = 2005.1$ K.

Figure 1 shows the results of calculation at $T_0 = 500$ K, $S_0 = 0.25$, $T^0 = 400$ K, $p^0 = 10^5$ Pa, and $k = 10^{-18}$ m² and the dimensionless temperature $T' = T/T_0$ (solid line), pressure $p' = p/p_0$ (dashed line), and saturation $S' = S/S_0$ (dash-and-dot line) as functions of the dimensionless self-similar variable $\xi' = \xi(kp_0/m\mu_{\mathrm{st}0})^{-1/2}$.

The results of numerical experiments indicate that the steam zone becomes narrower as the permeability increases. The steam zone degenerates at a particular value of the permeability, depending on the other parameters of the problem. Physically, this corresponds to the removal of steam–water mixture from the stratum. In the given case the mathematical model no longer contains moving saturation discontinuities and the phase transition takes place in the entire stratum. The boundary conditions at $x = 0$ change correspondingly. It is sufficient here to set the pressure of the steam–water mixture extracted from the stratum. The temperature of the mixture is determined from the condition of thermodynamic equilibrium and the saturation is the result of the solution of the problem. Figures 2 and 3 show examples of calculations at $k = 10^{-16}$ m² and $k = 10^{-14}$ m², respectively, for $T_0 = 500$ K, $S_0 = 0.5$, and $p^0 = 10^5$ Pa.

The results of calculations within the framework of the proposed model indicate that in the range of high permeabilities, which is of practical interest, solutions with moving saturation discontinuities are not realized. This means that heat carrier is extracted from the stratum in its initial qualitative state. The form of the saturation distributions obtained during the solution of the problem is determined in many ways by the distinctive features of the combined filtration of water and steam at various permeabilities and is evidence of processes of partial condensation and steam formation in the stratum.

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